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Principles of Programming Languages (CS-314)

Professor Ulrich Kremer

Section 02

Assignment 1

Solutions

1. Problem – Three simple rewrite systems
2. f(x) = x+1, x > 0

Rules for the rewrite system:

Rule 1: $| 🡪 |$

Rule 2: $# 🡪 |

Computation Example:

$|||#

* |$||#
* ||$|#
* |||$#
* ||||

1. f(x) = 2\*x, x > 0

Rules for the rewrite system:

Rule 1: $| 🡪 ||$

Rule 2: $|# 🡪 ||

Computation Example:

$|||#

* ||$||#
* ||||$|#
* ||||||

1. f(x,y) = x + y, x > 0, y > 0

Rules for the rewrite system:

Rule 1: $| 🡪 |$

Rule 2: $|& 🡪 |$

Rule 3: $|# 🡪 |

Computation Example:

$||&|||#

* |$|&|||#
* ||$|||#
* |||$||#
* ||||$|#
* |||||

1. Problem – A rewrite system for modulo 3 addition
2. Rewrite System for modulo 3 expressions

Rules for computation:

Rule 1: ((🡪 (

Rule 2: (1+ 🡪 1+

Rule 3: (0+ 🡪 0+

Rule 4: (2+ 🡪 2+

Rule 5: 1 + 0 🡪 1

Rule 6: 1 + 1 🡪 2

Rule 7: 1 + 2 🡪 0

Rule 8: 0 + 0 🡪 0

Rule 9: 0 + 1 🡪 1

Rule 10: 0 + 2 🡪 2

Rule 11: 2 + 0 🡪 2

Rule 12: 2 + 1 🡪 0

Rule 13: 2 + 2 🡪 1

Rule 14: 2) 🡪 2

Rule 15: 0) 🡪 0

Rule 16: 1) 🡪 1

1. Examples for computation:
2. ((1+2)+0)

* (1+2)+0)
* 1+2)+0)
* 0)+0)
* 0+0)
* 0)
* 0

1. (1+(2+2))

* 1+(2+2))
* 1+2+2))
* 0+2))
* 2))
* 2)
* 2

1. No. The rewrite system is not unique because we can proceed to the next step by using a different set of rules.

Example:

(1+(2+2))

* 1+(2+2))
* 1+2+2))

In the step above we can proceed with any rule that is, Rule 7 or Rule 13.

* 0+2))
* 2))
* 2)
* 2

1. Problem – Regular Expressions
2. Yes, we can eliminate r+ from our language and not reduce its expressiveness. The regular expression that we can use is r.r\*
3. Yes, we can eliminate r+ and r\* from our language and not reduce its expressiveness. The regular expressions that we can use are:
4. r|s (where s ∈ L(r) ∪ L(rr) ∪…….)
5. ε|r|s (where s ∈ L(r) ∪ L(rr) ∪…….)
6. Problem – Regular Expressions
7. Regular Expression: ε(digit)+(digit)+
8. Problem 5 – Regular Expressions
9. Strings of a’s and b’s.
10. Strings of 0’s and 1’s with even number of 1’s
11. Problem 6 – Regular Expressions